Optimal Channel Allocation in the CBRS Band with Shipborne Radar Incumbents

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***Abstract*—With the rollout of CBRS service, dynamic channel allocation for the PAL holders and GAA users remains a challenging problem in the spectrum community. To address this challenge, we develop a mathematical framework for optimal channel allocation for PAL holders and GAA users in a real-world setting. By exploiting the unique geographical characteristics associated with Navy shipborne radar incumbents along Virginia's east coast, we formulate an optimization problem for joint channel allocation of PAL holders and GAA users while meeting all interference threshold constraints for the incumbents and PAL holders. By leveraging several novel reformulation techniques, we show that the raw MINLP can be reformulated into a MILP with no approximation error. Through simulation experiments on real-world data for counties along Virginia's east coast, we show that our optimal solution can offer guaranteed interference protection to the incumbents and PAL holders while maximizing channel utilization for the GAA users.**

I. Introduction

The Citizens Broadband Radio Service (CBRS) is a 150 MHz frequency band between 3550 MHz and 3700 MHz in the US. Per FCC [1], CBRS is governed by a three-tiered spectrum sharing architecture to accommodate both federal and non-federal (commercial/civilian) users. At the highest tier are the incumbent (federal) users, at the middle tier are the Priority Access License (PAL) holders, and at the lowest tier are the General Authorized Access (GAA) users. Both PAL and GAA users are non-federal users. Operation and management of the three-tier users for spectrum sharing are centrally controlled by a dynamic spectrum access system (SAS) [2]–[4]. It was envisioned that under this new CBRS architecture, the existing 150 MHz band can be utilized much more efficiently.

Although the new CBRS band promises to offer much potential for spectrum sharing, there remains some significant challenges to fully harness its potential. A fundamental challenge is how to allocate channels to PAL and GAA users so that an array of interference constraints can be met and some desired objectives can be optimized. Specifically, the following questions must be addressed by a SAS when managing CBRS services: (i) How to allocate channels to PAL and GAA users in different counties so that the aggregate interference on a incumbent is kept under a target threshold? (ii) How to allocate channels so that the PAL holders' channel licenses can be guaranteed in the presence of GAA users? (iii) How to allocate channels so that the GAA users can maximize spectrum efficiency while keeping their interference on the PAL holders under control? (iv) How to ensure the aggregate mutual interference among the GAA users (after channel allocation) is not excessive so that it can offer a reasonable QoS? Although the FCC has set up some crude rules and regulations (e.g., interference thresholds for the incumbent and PAL holders [1]), there remains a lack of a rigorous mathematical framework for a SAS to meet these rules while offering optimal performance (w.r.t. some objective functions).

TABLE I
ACRONYM

Acronym	Full Name
CBRS	Citizen Broadband Radio Service
CBSD	Citizen Broadband Radio Service Device
DPA	Dynamic Protection Area
ESC	Environmental Sensing Capability
GAA	General Authorized Access
PAL	Priority Access License
PPA	PAL Protection Area
SAS	Spectrum Access System

Related Work Prior efforts to address these challenges remain limited. In [5], Souryal *et al.* studied the concept of "move list" (proposed in [6]) from which interferers (PAL or GAA users) will be subject to removal from a channel following the order of their interference levels should the incumbent in a DPA appear in that channel. Although simple, this naive approach suffers from a number of performance issues, such as disruption to PAL holders' operation on their licensed channels and efficiency in overall channel utilization.

A more efficient approach to address these challenges is to pursue optimized resource allocation [7]–[10]. In [7], Basnet *et al.* studied a power control problem among GAA users with the objective of maximizing capacity of GAA users operating on the same channel. However, the important problem of channel allocation for *both* PAL and GAA users was not addressed. In [8], Basnet *et al.* studied joint channel and power allocation for fixed and moving GAA users with a similar objective as in [7]. The authors proposed to decouple the channel allocation and power control problems and devised solutions to each. Again, the channel allocation problem for the PAL holders is not considered (jointly with that for GAA users). Further, the studies in [7] and [8] were only limited to one census tract. In [9], [10], the authors studied channel allocation for PAL and GAA users. In their proposed solutions, channel assignment for PAL and GAA users was done in two steps: with channel assignment for PAL done first, followed by GAA channel assignment. Clearly, such a decoupled approach cannot offer an optimal solution.

Scope and Contributions In this paper, we study optimal channel allocation for PAL and GAA users so that interference

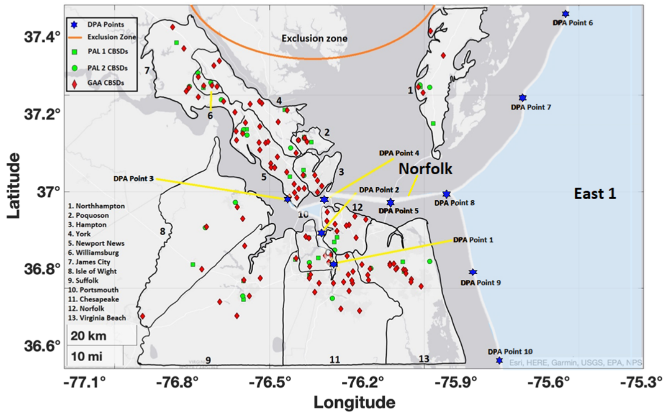

Fig. 1. Geographical topology for DPA points and 13 counties under the administrative domain of a SAS.

constraints on the incumbent and PAL holders can be met while a GAA-based utility function can be maximized. To maximize practical value of this research, we consider a real-world CBRS scenario where a SAS manages a group of counties along Virginia's coastal lines with Navy's shipborne radars as incumbents. The main contributions of this paper are summarized as follows:

- Based on FCC rules [1], we develop a mathematical framework for spectrum sharing of the three-tier services in the CBRS band. We characterize FCC rules on channel allocation and interference requirements through rigorous mathematical models. To protect the shipborne radar incumbents from interference, we exploit the unique geographical feature associated with coastal dynamic protection areas (DPAs) and represent them with finite discrete points.
- To ensure that our problem formulation can be solved efficiently, we perform reformulation on the original MINLP and obtain a new MILP. We show that our approach introduces no approximation errors and thus the optimal solution of MILP is also the optimal solution of the original MINLP.
- To validate our mathematical framework, we conduct simulation experiments with real-world CBRS map and DPAs along the east coast of Virginia, with Navy's shipborne radars as incumbents. We show that the optimal channel allocation results for the PAL and GAA users meet all the interference thresholds for the incumbent and PAL holders. Further, the computation time of our solution (on a desktop CPU) meets the timing requirement on a SAS.[1]

II. System Model

A. Topology Model

Table I lists the acronyms and Table II lists all notations in this paper. To concretize our discussion, consider Virginia's east coast as shown in Fig. 1. In this figure, we have a group of counties (each corresponding to a license area) under the management of a SAS. Denote $\mathcal{A}$ as the set of these counties. In this real world CBRS scenario, Navy ship-borne radars are the dominant incumbents. As expected, the exact locations for the Navy radars cannot be disclosed to the public. Instead, Environmental Sensing Capability (ESC) sensors are deployed along the coast to detect the presence of these radars [11]. Specifically, the coastal area on the water is divided into DPAs, which are pre-defined protection areas for the Navy radars to operate with strictly-controlled interference from inland PAL and GAA users (see DPAs EAST1 and Norfolk in Fig. 1). Denote $\mathcal{I}$ as the set of DPAs along the coast. Upon detecting the presence of incumbents in a DPA, an ESC sensor will measure the signal strength of the incumbents and report this information to the SAS.

In our shipborne incumbent scenario, all PAL and GAA users can only be present inland within the counties. Denote $\mathcal{P}_j$ as the set of PAL holders in county j and $\mathcal{B}_{i|j}$ as the set of CBSDs of PAL holder i ($i \in \mathcal{P}_j$) that is present in county

[1]Per FCC [1], the maximum allowed time for SAS to re-allocate PAL/GAA channels (to meet incumbent users' needs) is 5 minutes. We use this time as our benchmark for the maximum allowed computation time.

TABLE II
Notations

Symbol	Definition
General notations for CBRS	
$\mathcal{A}$	A set of CBRS license areas (counties)
$\mathcal{F}$	A set of 15 channels in CBRS band
Notations for PAL	
$\mathcal{F}_P$	A set of 10 channels in $\mathcal{F}$ that can be used for PAL
$\mathcal{P}_j$	A set of PAL holders in county j
$\mathcal{B}_{i\|j}$	A set of CBSDs of PAL holder i in county j
$x^c_{i\|j}$	A binary variable that indicates whether or not channel c is assigned to PAL holder i in county j
$L^{\mathrm{P}}_{i\|j}$	Total number of licenses held by PAL holder i in county j
$\gamma^c_{(i,b)\|j,(k,e)\|l}$	Received signal strength on channel c at a location in the PPA of CBSD $e \in \mathcal{B}_{k\|l}$ that is closest to CBSD $b \in \mathcal{B}_{i\|j}$
$\delta^c_{n\|j,(k,e)\|l}$	Received signal strength on channel c at a location in the PPA of $e \in \mathcal{B}_{k\|l}$ that is closest to GAA user n in county j
T^{P}_i	An interference threshold for any point within the PPAs of PAL holder i's CBSDs
Notations for GAA	
$\mathcal{G}_j$	A set of GAA users in county j
$y^c_{n\|j}$	A binary variable that indicates whether or not channel c is assigned to GAA user n in county j
$L^{\mathrm{G}}_{n\|j}$	Maximum number of channels that can be assigned to GAA user n in county j
$D_{n,g}$	Distance between GAA user n and GAA user g
R_n	Radius of GAA user n's coverage area
Notations for DPAs containing incumbents	
$\mathcal{I}$	A set of DPAs for incumbents along the coast
Z^c_m	A binary parameter that indicates whether or not channel c is used by any active incumbent in DPA $m \in \mathcal{I}$
$\alpha^c_{(i,b)\|j,m}$	Received signal strength on channel c at a location on the boundary of DPA $m \in \mathcal{I}$ that is closest to CBSD $b \in \mathcal{B}_{i\|j}$
$\beta^c_{n\|j,m}$	Received signal strength on channel c at a location on the boundary of DPA $m \in \mathcal{I}$ that is closest to GAA user n in county j
T^{I}_m	An interference threshold for all locations within DPA $m \in \mathcal{I}$

j. Note that the set $\mathcal{B}_{i|j}$ of CBSD boxes from PAL holder i in county j are all assigned to the same set of channels.

Denote $\mathcal{G}_j$ as the set of GAA users in the CBRS in county j. Unlike PAL, where one PAL holder i may have a set $\mathcal{B}_{i|j}$ of CBSD boxes operating in the same county j, we assume that one GAA user corresponds to only one CBSD box. Therefore, when there is no ambiguity, we use the terms GAA user and GAA CBSD interchangeably throughout the paper.

B. Channel Allocation Rules and Constraints

PAL Holders Denote $\mathcal{F} = \{1, 2, \cdots, 15\}$ as the set of 15 CBRS channels, each occupying 10 MHz bandwidth. Denote $\mathcal{F}_P = \{1, 2, \cdots, 10\}$ as a subset of $\mathcal{F}$ containing the first 10 CBRS channels that can be licensed to PAL holders [1]. Further, no more than 7 channels from $\mathcal{F}_P$ can be used for PAL holders.

Denote $L^{\mathrm{P}}_{i|j}$ as the number of channel licenses assigned to PAL holder i in county j. $L^{\mathrm{P}}_{i|j}$ can be awarded only during the bidding process and cannot exceed 4 in any county $j \in \mathcal{A}$ for $i \in \mathcal{P}_j$.

In this paper, we relax the channel contiguity requirements for channels assigned to a PAL holder in the same county as well as geographic contiguity requirement for channels assigned to a PAL holder across neighboring counties. Such a relaxation will enlarge optimization space for channel allocation and likely offer better performance in spectrum utilization. However, we will leave the detailed comparison study in a future paper.

Denote $x^c_{i|j}$ as a binary variable indicating whether or not channel c is assigned to PAL holder i in county j, i.e.,

$$x^c_{i|j} = \begin{cases} 1, & \text{if channel } c \text{ is assigned to PAL holder } i \text{ in county } j \\ 0, & \text{otherwise.} \end{cases}$$

Since each channel $c \in \mathcal{F}_P$ in county $j \in \mathcal{A}$ can only be assigned to at most one PAL holder,[2] we have:

$$\sum_{i \in \mathcal{P}_j} x^c_{i|j} \le 1 \qquad (c \in \mathcal{F}_P,\ j \in \mathcal{A}), \tag{1}$$

and

$$\sum_{c \in \mathcal{F}_P} x^c_{i|j} = L^{\mathrm{P}}_{i|j} \qquad (i \in \mathcal{P}_j,\ j \in \mathcal{A}). \tag{2}$$

For a PAL holder k in a county l, recall that $\mathcal{B}_{k|l}$ is the set of CBSDs that operate on the same set of licensed channels. Per rules set forth by FCC in [1], for any CBSD $e \in \mathcal{B}_{k|l}$, its received interference on any point in its PPA should not exceed a threshold of -80 dBm/10MHz. Such interference only includes those that come from PAL CBSDs and GAA users operating on the same channel. To model this constraint, denote $\gamma^c_{(i,b)|j,(k,e)|l}$ as the received signal strength on channel c at the location within the PPA of PAL CBSD $e \in \mathcal{B}_{k|l}$ that is closest to PAL CBSD $b \in \mathcal{B}_{i|j}$.

[2]But the same channel may be assigned also to a GAA user, if the latter meets the interference constraint for PAL in (3).

For GAA users, denote $y^c_{n|j}$ as a binary variable that indicates whether or not channel c is assigned to GAA user n in county j, i.e.,

$$y^c_{n|j} = \begin{cases} 1, & \text{if channel } c \text{ is assigned to GAA user } n \text{ in county } j \\ 0, & \text{otherwise,} \end{cases}$$

Denote $\delta^c_{n|j,(k,e)|l}$ as the received signal strength on channel c at the location within the PPA of PAL CBSD $e \in \mathcal{B}_{k|l}$ that is closest to GAA user n in county j. Then we have the following interference constraint:

$$\begin{aligned} & x^c_{k|l} \sum_{j \in \mathcal{A}, j \ne l} \sum_{i \in \mathcal{P}_j, i \ne k} \sum_{b \in \mathcal{B}_{i|j}} x^c_{i|j} \cdot \gamma^c_{(i,b)|j,(k,e)|l} + \\ & \qquad x^c_{k|l} \sum_{j \in \mathcal{A}} \sum_{n \in \mathcal{G}_j} y^c_{n|j} \cdot \delta^c_{n|j,(k,e)|l} \le T^{\mathrm{P}}_k \\ & \qquad\qquad (c \in \mathcal{F}_P,\ l \in \mathcal{A},\ k \in \mathcal{P}_l,\ e \in \mathcal{B}_{k|l}), \end{aligned} \tag{3}$$

where T^{P}_k denotes the interference threshold for the PPA of PAL CBSD $e \in \mathcal{B}_{k|l}$. In (3), the first term represents the aggregate interference from all (other than k) PAL holders' CBSDs from all counties (other than l) that produced interference to CBSD $e \in \mathcal{B}_{k|l}$ on channel c; the second term represents the aggregate interference from all GAA users from all counties (including l) that produce interference to CBSD $e \in \mathcal{B}_{k|l}$ on channel c.

GAA Users Denote $L^{\mathrm{G}}_{n|j}$ as the maximum number of channels that can be assigned to GAA user n in county j. Since SAS is not required to guarantee $L^{\mathrm{G}}_{n|j}$ channels to GAA user n (as it does to PAL holders), it may assign any number between 0 and $L^{\mathrm{G}}_{n|j}$ to this GAA user. We have:

$$\sum_{c \in \mathcal{F}} y^c_{n|j} \le L^{\mathrm{G}}_{n|j} \qquad (j \in \mathcal{A},\ n \in \mathcal{G}_j). \tag{4}$$

Per FCC [1], the GAA users are not protected from (and thus must accept) interference from the incumbents and PAL holders. GAA users can use any channel of the CBRS band as long as its interference to the two upper tiers is below their respective interference thresholds. We have considered GAA users' interference to PAL CBSDs in constraint (3) and will consider their interference to incumbents in constraint (6) later in this section.

Although GAA users are not protected from interference from PAL holders and the incumbents, it would still be wise to exercise some level of co-channel interference control from other GAA users. Otherwise, excessive mutual interference among GAA users can easily make such service unusable and defeat its practical utility. Therefore, we introduce the following constraint:

$$\begin{aligned} & D_{n,g} \ge y^c_{n|j} \cdot y^c_{g|j} \cdot (R_n + R_g) \\ & \qquad (c \in \mathcal{F},\ j \in \mathcal{A},\ g \in \mathcal{G}_j,\ n \in \mathcal{G}_j, n \ne g), \end{aligned} \tag{5}$$

where $D_{n,g}$ denotes the distance between GAA users n and g, R_n and R_g denote transmission ranges of GAA user n and g, respectively. Constraint (5) says that when two GAA CBSDs, n and g, are using the same channel c (i.e., when $y^c_{n|j} = 1$

and $y^c_{g|j} = 1$), there must be sufficient geographical separation between them to keep co-channel interference under control.

DPAs for Incumbents Denote Z^c_m as a binary parameter to indicate whether or not channel c is used by any active incumbent in DPA $m \in \mathcal{I}$, i.e.,

$$Z^c_m = \begin{cases} 1, & \text{if channel } c \in \mathcal{F} \text{ is used by an active incumbent located in DPA } m \in \mathcal{I}; \\ 0, & \text{otherwise}. \end{cases}$$

Note that the values of Z^c_m's are known *a priori,* which are determined through the reports from ESC sensors along the coast.

To protect the active incumbents in a DPA, the aggregate interference from all inland PAL holders' CBSDs and GAA users operating on the same channel must not exceed a threshold (as perceived by any point in the DPA). Specifically, NTIA has defined that such aggregate interference must not exceed -144 dBm/10MHz [12].

To develop a mathematical constraint for this requirement, it is sufficient to consider the west boundary of DPA East1 and the boundary of Norfolk. For each boundry, we propose to discretize it into a finite number of points, as shown in Fig. 1. In this figure, the west boundary of DPA East1 is represented by 5 discrete points (6 to 10) while DPA Norfolk is represented by another 5 points (1 to 5). For each DPA, we will formulate a constraint using a (conservative) worst case estimate of distance between an interferer's CBSD and the DPA. For example, with respect to a specific inland PAL CBSD or GAA user, its worst case interference to the DPA can be approximated by its interference to a closest point among the 5 points for that DPA.

With our discretization and worst case approximation, the interference protection to DPA m from PAL and GAA users can be modeled with the following mathematical constraint:

$$Z^c_m \sum_{j\in\mathcal{A}} \sum_{i\in\mathcal{P}_j} \sum_{b\in\mathcal{B}_{i|j}} x^c_{i|j} \cdot \alpha^c_{(i,b)|j,m} + Z^c_m \sum_{j\in\mathcal{A}} \sum_{n\in\mathcal{G}_j} y^c_{n|j} \cdot \beta^c_{n|j,m} \leq T^{\mathrm{I}}_m \quad (c \in \mathcal{F},\ m \in \mathcal{I}). \tag{6}$$

where $\alpha^c_{(i,b)|j,m}$ denotes received signal strength on channel c at a DPA point on the boundary of DPA $m \in \mathcal{I}$ that is closest to PAL CBSD $b \in \mathcal{B}_{i|j}$; $\beta^c_{n|j,m}$ denotes received signal strength on channel c at a DPA point on the boundary of DPA $m \in \mathcal{I}$ that is closest to GAA user n in county j, and T^{I}_m denotes the interference threshold for any location within DPA $m \in \mathcal{I}$.

III. Problem Formulation

In the last section, we developed a set of constraints for the three-tiered users in CBSD. These constraints, when stated jointly, must be feasible. Otherwise, there must be a constraint violation during the PAL auction phase (i.e., awarding more channel licenses to some PAL holders than what are available). But this must not be allowed during the auction phase.

Although the set of constraints in the last section is always feasible, there are many possibilities (feasible solutions) for channel assignment (i.e., $x^c_{i|j}$'s and $y^c_{n|j}$'s). So the question here is what kind of channel assignment is most desirable. The answer to this question depends on our objective function.

A number of objective functions may be considered. In this paper, we aim to maximize total channel utility across all GAA users across all counties, i.e., $\max \sum_{j\in\mathcal{A}} \sum_{n\in\mathcal{G}_j} \ln(1 + \sum_{c\in\mathcal{F}} y^c_{n|j})$. Note that we need to add "1" inside the "ln" function to ensue the "ln" function is non-negative. We now have the following optimization problem.

$$\begin{aligned} \text{OPT}: \ \max \ & \sum_{j\in\mathcal{A}} \sum_{n\in\mathcal{G}_j} \ln(1 + \sum_{c\in\mathcal{F}} y^c_{n|j}) \\ \text{s.t.} \ & \text{PAL channel assignment constraints (1), (2)}, \\ & \text{PAL interference protection constraint (3)}, \\ & \text{GAA channel assignment constraint (4)}, \\ & \text{GAA mutual interference control constraint (5)}, \\ & \text{Incumbent interference protection constraint (6)}, \\ & x^c_{i|j} \in \{0,1\} \qquad (c \in \mathcal{F}_P,\ j \in \mathcal{A},\ i \in \mathcal{P}_j), \\ & y^c_{n|j} \in \{0,1\} \qquad (c \in \mathcal{F},\ j \in \mathcal{A},\ n \in \mathcal{G}_j). \end{aligned}$$

Problem OPT is a Mixed Integer Non-Linear Program (MINLP), with nonlinear terms in the objective function, constraint (3), and constraint (5). In the rest of this section, we show how to reformulate these terms.

A. Reformulation

Objective function We can reformulate the log function into a series of linear constraints and show this reformulation is exact for our problem. By "exact" we mean the optimization space stays the same after reformulation and the optimal objective value remains unchanged before and after reformulation.

Specifically, since $y^c_{n|j}$'s are binary variables and the maximum of $\sum_{c\in\mathcal{F}} y^c_{n|j}$ is typically a small number, we can use a series of line segments (i.e., linear constraints) to serve as approximation of the log function. Since the hardware of a GAA's CBSD is identical to that of a PAL CBSD, the maximum of channels that a GAA user can have is 4, i.e., $\sum_{c\in\mathcal{F}} y^c_{n|j} \leq 4$. So we can define an integer variable $s_{n|j}$ as follows:

$$s_{n|j} = \sum_{c\in\mathcal{F}} y^c_{n|j} \qquad (j \in \mathcal{A}, n \in \mathcal{G}_j)\ . \tag{7}$$

Define $w_{n|j}$ as:

$$w_{n|j} = \ln(1 + s_{n|j}) \qquad (j \in \mathcal{A}, n \in \mathcal{G}_j)\ . \tag{8}$$

Based on (7) and (8), $(s_{n|j}, w_{n|j})$ can only take the following 5 points: $(0,0), (1,\ln(2)), (2,\ln(3)), (3,\ln(4)), (4,\ln(5))$, as shown in Fig. 2 (red circles). By connecting these two adjacent points from these five points sequentially, we obtain four line segments (shown as blue lines in Fig. 2). The four line segments represent a linear relaxation of the five points and can be written as the following four linear constraints:

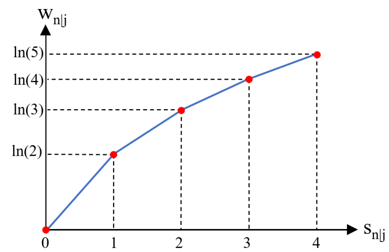

Fig. 2. Reformulation of log term $\ln(1 + \sum_{c\in\mathcal{F}} y^c_{n|j})$

$$w_{n|j} \leq \ln(2)(s_{n|j}) \tag{9a}$$
$$w_{n|j} \leq (\ln(3) - \ln(2))(s_{n|j} - 1) + \ln(2) \tag{9b}$$
$$w_{n|j} \leq (\ln(4) - \ln(3))(s_{n|j} - 2) + \ln(3) \tag{9c}$$
$$w_{n|j} \leq (\ln(5) - \ln(4))(s_{n|j} - 3) + \ln(4) \tag{9d}$$
$$(j \in \mathcal{A}, n \in \mathcal{G}_j) \ .$$

With the definitions of $s_{n|j}$ and $w_{n|j}$, the objective function of OPT now becomes $\max \sum_{j\in\mathcal{A}} \sum_{n\in\mathcal{G}_j} w_{n|j}$.

Constraints (3) and (5) The nonlinear terms in constraint (3) are the bilinear terms $x^c_{k|l} \cdot x^c_{i|j}$ and $x^c_{k|l} \cdot y^c_{n|j}$. We propose to use RLT to linearize both [13]. RLT is a powerful linearization tool that provides tight linear relaxations for any monomial term (i.e., product of polynomial terms). The essence of RLT is to replace monomial terms with auxiliary variables and add constraints for these auxiliary variables based on their relations with the original decision variables. For a bilinear term consisting of two binary decision variables, RLT introduces no relaxation error and thus is a perfect tool for this reformulation.

To apply RLT, we introduce a new set of auxiliary variables $X^c_{i|j,k|l}$, defined as

$$X^c_{i|j,k|l} = x^c_{i|j} \cdot x^c_{k|l} \ . \tag{10}$$

For the additional constraints regarding $X^c_{i|j,k|l}$, we derive them based on the upper bounds and lower bounds of $x^c_{i|j}$ and $x^c_{k|l}$. Since $x^c_{i|j}$ and $x^c_{k|l}$ are binary variables, their upper bounds and lower bounds are 0 and 1 respectively. Since $0 \leq x^c_{i|j} \leq 1$, and $0 \leq x^c_{k|l} \leq 1$, we have:

$$\begin{aligned}
(x^c_{i|j} - 0) \cdot (x^c_{k|l} - 0) &\geq 0 \ , \\
(x^c_{i|j} - 0) \cdot (1 - x^c_{k|l}) &\leq 0 \ , \\
(1 - x^c_{i|j}) \cdot (x^c_{k|l} - 0) &\leq 0 \ , \\
(1 - x^c_{i|j}) \cdot (1 - x^c_{k|l}) &\geq 0 \ ,
\end{aligned}$$

which are equivalent to:

$$\begin{aligned}
x^c_{i|j}x^c_{k|l} \geq 0 \ , &\quad x^c_{i|j} - x^c_{i|j}x^c_{k|l} \geq 0 \ , \\
x^c_{k|l} - x^c_{i|j}x^c_{k|l} \geq 0 \ , &\quad x^c_{i|j} + x^c_{k|l} - x^c_{i|j}x^c_{k|l} \leq 1 \ .
\end{aligned} \tag{11}$$

Substituting (10) into (11), we obtain:

$$\begin{aligned}
X^c_{i|j,k|l} \geq 0 \ , &\quad x^c_{i|j} - X^c_{i|j,k|l} \geq 0, \\
x^c_{k|l} - X^c_{i|j,k|l} \geq 0 \ , &\quad x^c_{i|j} + x^c_{k|l} - X^c_{i|j,k|l} \leq 1, \\
(c \in \mathcal{F}_P, \ j \in \mathcal{A}, i \in \mathcal{P}_j&, l \in \mathcal{A}, k \in \mathcal{P}_l, i \neq k) \ .
\end{aligned} \tag{12}$$

Constraints (12) are linear constraints associated with $X^c_{i|j,k|l}$ based on RLT. It is easy to verify that they are equivalent to constraint (10) since $x^c_{i|j}$ and $x^c_{k|l}$ are binary decision variables. Following the same token, we can define

$$V^c_{n|j,k|l} = y^c_{n|j}x^c_{k|l} \ , \tag{13}$$

and use the same RLT procedure to obtain the following linear constraints:

$$\begin{aligned}
V^c_{n|j,k|l} \geq 0 \ , &\quad y^c_{n|j} - V^c_{n|j,k|l} \geq 0, \\
x^c_{k|l} - V^c_{n|j,k|l} \geq 0 \ , &\quad y^c_{n|j} + x^c_{k|l} - V^c_{n|j,k|l} \leq 1, \\
(c \in \mathcal{F}_P, \ j \in \mathcal{A}, n \in \mathcal{G}_j&, l \in \mathcal{A}, k \in \mathcal{P}_l) \ .
\end{aligned} \tag{14}$$

Substituting (10) and (13) into (3), we have

$$\begin{aligned}
&\sum_{j\in\mathcal{A}, j\neq l} \sum_{i\in\mathcal{P}_j, i\neq k} \sum_{b\in\mathcal{B}_{i|j}} X^c_{i|j,k|l}\gamma^c_{(i,b)|j,(k,e)|l} + \\
&\qquad \sum_{j\in\mathcal{A}} \sum_{n\in\mathcal{G}_j} V^c_{n|j,k|l}\delta^c_{n|j,(k,e)|l} \leq T^{\mathrm{P}}_k \\
&\qquad\qquad (c \in \mathcal{F}_P, \ l \in \mathcal{A}, \ k \in \mathcal{P}_l, \ e \in \mathcal{B}_{k|l}).
\end{aligned} \tag{15}$$

Now constraint (15) is linear since $\gamma^c_{(i,b)|j,(k,e)|l}$ and $\delta^c_{n|j,(k,e)|l}$ are constants. So we can replace nonlinear constraint (3) in OPT with linear constraints (12), (14), and (15).

For constraint (5), we again apply the same RLT procedure by replacing $y^c_{n|j}y^c_{g|j}$ with a new variable and adding a set of linear constraints. We define

$$Y^c_{n|j,g|j} = y^c_{n|j} \cdot y^c_{g|j} \tag{16}$$

and apply RLT for the bilinear terms $y^c_{n|j} \cdot y^c_{g|j}$ in constraint (5). We obtain the following 4 linear constraints:

$$\begin{aligned}
Y^c_{n|j,g|j} \geq 0 \ , &\quad y^c_{n|j} - Y^c_{n|j,g|j} \geq 0, \\
y^c_{g|j} - Y^c_{n|j,g|j} \geq 0 \ , &\quad y^c_{n|j} + y^c_{g|j} - Y^c_{n|j,g|j} \leq 1, \\
(c \in \mathcal{F} \ , j \in \mathcal{A}, \ g &\in \mathcal{G}_j, n \in \mathcal{G}_j, n \neq g) \ .
\end{aligned} \tag{17}$$

Substituting (16) in (5), we have:

$$\begin{aligned}
D_{n,g} \geq & Y^c_{n|j,g|j}(R_n + R_g) \\
&(c \in \mathcal{F}, j \in \mathcal{A}, \ g \in \mathcal{G}_j, n \in \mathcal{G}_j, n \neq g) \ .
\end{aligned} \tag{18}$$

So we can replace constraint (5) in OPT with constraints (17) and (18).

B. A Summary of Reformulation

With the above linearizations for the objective function, constraint (3), and constraint (5) in OPT, we have the following new problem formulation (denoted as OPT-R):

$$\begin{aligned}
\text{OPT-R}: \ \max & \sum_{j\in\mathcal{A}}\sum_{n\in\mathcal{G}_j} w_{n|j} \\
\text{s.t.} \ & \text{Linearization of log function } (7),(9), \\
& \text{PAL channel assignment constraints } (1),(2), \\
& \text{Linearized PAL interference protection} \\
& \text{constraints } (12),(14),(15), \\
& \text{GAA channel assignment constraints } (4), \\
& \text{Linearized GAA mutual interference control} \\
& \text{constraints } (17),(18), \\
& \text{Incumbent interference protection constraints } (6), \\
& x^c_{i|j}\in\{0,1\} \qquad (c\in\mathcal{F}_P,\ j\in\mathcal{A},\ i\in\mathcal{P}_j), \\
& y^c_{n|j}\in\{0,1\} \qquad (c\in\mathcal{F},\ j\in\mathcal{A},\ n\in\mathcal{G}_j), \\
& w_{n|j}\geq 0 \qquad (j\in\mathcal{A},\ n\in\mathcal{G}_j), \\
& s_{n|j}\geq 0 \qquad (j\in\mathcal{A},\ n\in\mathcal{G}_j), \\
& X^c_{i|j,k|l}\in\{0,1\}, \\
& (c\in\mathcal{F}_P,\ j\in\mathcal{A}, i\in\mathcal{P}_j, l\in\mathcal{A}, k\in\mathcal{P}_l, i\neq k), \\
& V^c_{n|j,k|l}\in\{0,1\}, \\
& (c\in\mathcal{F}_P,\ j\in\mathcal{A}, n\in\mathcal{G}_j, l\in\mathcal{A}, k\in\mathcal{P}_l), \\
& Y^c_{n|j,g|j}\in\{0,1\}, \\
& (c\in\mathcal{F}\ , j\in\mathcal{A},\ g\in\mathcal{G}_j, n\in\mathcal{G}_j, n\neq g).
\end{aligned}$$

Equivalence OPT-R is an MILP. We now show that an optimal solution to OPT-R is also an optimal solution to OPT. First, the objective functions of OPT-R and OPT are identical due to $\sum_{j\in\mathcal{A}}\sum_{n\in\mathcal{G}_j} w_{n|j} = \sum_{j\in\mathcal{A}}\sum_{n\in\mathcal{G}_j}\ln(1+\sum_{c\in\mathcal{F}} y^c_{n|j})$. Second, it is easy to see that an optimal solution to OPT-R is also a feasible solution to OPT. This is because that an optimal solution to OPT-R, based on our reformulation process, satisfies all the constraints for OPT. Third, it is also easy to show that the optimal solution to OPT is a feasible solution to OPT-R based on our reformulation, since it will satisfy all the constraints for OPT-R. Therefore, as far as the optimal objective value is concerned, it is sufficient to find an optimal solution to OPT-R.

Being a MILP, OPT-R can be solved by commercial solvers such as CPLEX and Gurobi. In our simulation experiment in the next section, we show that the computation time of solving OPT-R for our real world CBRS scenario is under 5 minutes, which meets the timing requirement set forth for a SAS [1].

IV. Simulation Experiments

In this section, we conduct a numerical study for OPT-R, using topology and CBRS settings from the real world.

A. Topology and CBRS Parameter Settings

We consider 13 counties on the east coast of Virginia that are neighboring of DPAs Norfolk and East1. Note that the 13 counties are all outside (below) the exclusion zone in Fig. 1.[3]

[3]Per [6], exclusion zone is an area where additional restrictions are imposed on CBSD operations. NTIA has made the KML files about exclusion zones publicly available on its website [12], [15]. It is reasonable to assume that counties inside an exclusion zone will be handled separately.

For incumbents, we consider Norfolk and East1 DPAs as shown in Fig. 1. As shown in Fig. 1, we use 5 DPA points in Norfolk and 5 DPA points on the west boarder of East1 to represent each DPA respectively. The coordinates for these DPA points (in the format of latitude and longitude) are given as follows:

DPA points representing Norfolk:
1.{36.813768, -76.294250}, 2.{36.894569, -76.333859}, 3.{36.982050, -76.440279}, 4.{36.973018, -76.109891}, 5.{36.973018, -76.109891},
DPA points representing East1:
6.{37.460269, -75.544060}, 7.{37.244850, -75.682051}, 8.{36.995809, -75.927450}, 9.{36.791528, -75.843339}, 10.{36.563209, -75.756679}.

We assume the incumbents in the DPAs are present on CBRS channels 6, 7, and 8, i.e., $Z^6_m = 1$, $Z^7_m = 1$, $Z^8_m = 1$. The interference threshold for the incumbents on all these channels in the DPAs is $T^{\text{I}}_m = -144$ dBm/10MHz [12].

For the 13 inland counties, we assume there are two PAL holders present in each county, i.e., $\mathcal{P}_j = \{1, 2\}$. Each PAL holder has a set of CBSDs $\mathcal{B}_{i|j}$ deployed in each county j. The locations for each PAL holder's CBSDs are selected randomly within each county (correlated with population density) and are listed in Table III (in the format of latitude and longitude). For a PAL holder in a county, the number of its licenses is randomly generated between 1 and 4 and that the sum of licenses assigned to all PAL holders in a county is no more than 7. We assume all PAL CBSDs are category B, with a transmission power of 47 dBm/10MHz [1]. We employ pathloss model $PL(dB) = 128.1 + 37.6 \times \log_{10}(d)$ to calculate the received power from a transmitter over a distance d (in Km) [14]. Based on a CBSD's transmission power and the pathloss model, we can draw the PPA (a disk) for each PAL CBSD w.r.t a contour of power level -96 dBm/10MHz. The radius of a PPA is 2.491 Km (with the PAL's CBSD at its center). The allowed interference threshold in a PPA is $T^{\text{P}}_i = -80$ dBm/10MHz [1].

For GAA users, the locations of CBSDs in each county, as well as each CBSD's channel demand are listed in Table IV. The locations for GAA users are randomly selected within their respective counties (correlated to the population density) and the channel demand for each GAA user is randomly generated between 1 and 4. Similar to the PAL CBSDs, we assume all GAA CBSDs are category B, with a transmission power level of 47 dBm/10MHz. With the same contour of power level -96 dBm/10MHz and the same pathloss model used in PPA calculation, the transmission range (disk) of a GAA user is $R_n = 2.491$ Km.

B. Results

With the topology and CBRS parameter settings in Section IV-A, we present the solutions to OPT-R. All optimization problems are solved on a desktop computer—MacPro (2013) with 3 GHz 8-Core Intel Xeon E5 processor and 64 GB memory. We use Gurobi version 9.12 with CVX version 2.2 implemented in MATLAB R2021a to solve OPT-R.

TABLE III

PAL HOLDERS, CBSD LOCATIONS, LICENSES, AND SOLUTIONS OF CHANNEL ASSIGNMENT

County	1		2		3		4		5		6		7	
PAL holder	1	2	1	2	1	2	1	2	1	2	1	2	1	2
CBSD 1 location	{37.177323, -75.970480}	{37.276303, -76.012992}	{37.141530, -76.387953}	{37.138860, -76.387291}	{37.056313, -76.390138}	{37.042386, -76.388150}	{37.211766, -76.451027}	{37.113845, -76.431727}	{37.154085, -76.586840}	{37.161176, -76.583920}	{37.269244, -76.713815}	{37.080506, -76.689002}	{37.385214, -76.799968}	{37.271720, -76.758525}
CBSD 2 location	{37.270626, -76.012935}	{37.270047, -75.981610}	{37.128932, -76.365247}	{37.133396, -76.406430}	{37.039529, -76.434108}				{37.161758, -76.573339}	{37.147249, -76.572925}		{37.227398, -76.731162}		{37.238016, -76.655949}
Licenses	2	2	2	2	4	1	3	2	4	2	2	3	2	2
Channel	2,10	1,4	2,5	3,9	1,4,9,10	3	1,2,4	3,5	1,2,4,5	3,9	2,4	1,9,10	7,8	1,2
County	8		9		10		11		12		13			
PAL holder	1	2	1	2	1	2	1	2	1	2	1	2		
CBSD 1 location	{36.811722, -76.747829}	{36.972899, -76.610515}	{36.721565, -76.583553}	{36.729794, -76.586899}	{36.881306, -76.372402}	{36.816062, -76.307244}	{36.784180, -76.370982}	{36.724449, -76.296420}	{36.882713, -76.282134}	{36.849330, -76.289377}	{36.818632, -76.068517}	{36.801619, -76.172511}		
CBSD 2 location		{36.907787, -76.707530 }	{36.769728, -76.583669}		{36.828897, -76.342609}	{36.816958, -76.371151}	{36.824855, -76.416310}		{36.869959, -76.290342}		{36.793292, -76.065166}	{36.820032, -75.982165}		
Licenses	1	3	1	4	2	2	2	1	2	4	2	3		
Channel	9	2,3,5	10	2,4,5,9	5,9	2,3	3,9	4	3,4	1,2,9,10	2,3	4,5,9		

TABLE IV

GAA USERS' LOCATIONS, CHANNEL DEMANDS, AND SOLUTIONS OF CHANNEL ASSIGNMENT

County	1			2			3			4			5			6			7		
GAA	Location	Demand	Channels	Location	Demand	Channels	Location	Demand	Channels	Location	Demand	Channels	Location	Demand	Channels	Location	Demand	Channels	Location	Demand	Channels
1	{37.415468, -75.979523}	3	11,13,15	{37.129402, -76.379023}	3	12,13,14	{37.051484, -76.444348}	3	13,14,15	{37.337709, -76.661846}	3	13,14,15	{37.126961, -76.512230}	3	4,9,12	{37.296869, -76.729602}	3	5,11,12	{37.425609, -76.812708}	2	7,8
2	{37.271522, -76.018555}	4	11,12,13,15	{37.139943, -76.386314}	1	4	{37.043120, -76.391269}	4	1,2,9,11	{37.325906, -76.679980}	1	9	{37.127987, -76.532945}	2	1,15	{37.274210, -76.712464}	2	14,15	{37.370587, -76.776300}	3	1,11,15
3	{37.256361, -76.003451}	1	14	{37.133613, -76.406116}	2	10,15	{37.043272, -76.353228}	4	5,10	{37.272201, -76.669700}	4	5,11,13,14	{37.150316, -76.585846}	3	3,9,11	{37.274875, -76.685229}	2	10,13	{37.269591, -76.759877}	3	1,3,15
4	{37.352613, -75.938168}	4	11,13,14,15				{37.019980, -76.420547}	1	5	{37.226609, -76.562114}	4	1,4,13,15	{37.168628, -76.530550}	1	13				{37.260136, -76.768400}	3	12,13,14
5							{37.009404, -76.386963}	4	3,4,13,15	{37.229210, -76.524329}	3	11,12,14	{36.998532, -76.419544}	2	11,13				{37.245235, -76.730314}	2	10,11
6							{36.999072, -76.344692}	4	1,2,9,11	{37.233950, -76.532690}	3	2,3,5	{37.133361, -76.605951}	1	15				{37.234186, -76.648681}	3	5,11,15
7							{37.028875, -76.345183}	2	12,14	{37.181070, -76.471590}	3	10,13,14	{37.179345, -76.577736}	1	12				{37.205981, -76.611323}	3	1,10,14
8							{37.015473, -76.331208}	3	3,13,15	{37.210741, -76.442619}	2	1,15	{37.130075, -76.504505}	3	3,10,11						
9							{37.032040, -76.333296}	1	4				{37.073562,-76.496198}	3	10,11,13						
10							{37.007983, -76.404826}	2	10,12				{37.063718, -76.494798}	2	1,2						
11							{36.985370, -76.411089}	4	1,2,9,14				{37.063346, -76.482979}	4	3,4,5,14						
12							{36.986652, -76.435759}	3	3,4,13				{37.090387, -76.473828}	1	9						
13							{37.100778, -76.413305}	4	2,11,12,13				{37.062840, -76.483138}	2	12,15						
14													{37.108376, -76.530391}	2	2,14						
15													{37.156604, -76.607652}	3	10,13,14						

County	8			9			10			11			12			13		
GAA	Location	Demand	Channels	Location	Demand	Channels	Location	Demand	Channels	Location	Demand	Channels	Location	Demand	Channels	Location	Demand	Channels
1	{36.961074, -76.604177}	1	15	{36.730488, -76.564463}	3	11,12,13	{36.884489, -76.382210}	4	3,4,5,11	{36.763619, -76.231418}	4	11,12,15	{36.928539, -76.188754}	4	4,5,11,15	{36.812054, -76.201236}	4	1,3,11,14
2	{36.931124, -76.586982}	1	11	{36.678645, -76.605378}	2	11,13	{36.880300, -76.372321}	4	12,13,14,15	{36.743049, -76.242903}	2	1,14	{36.937157, -76.223752}	2	13,14	{36.801225, -76.174339}	1	9
3	{36.909366, -76.703523}	2	1,13	{36.715017, -76.660964}	3	1,11,15	{36.835251, -76.319687}	3	4,11,12	{36.766686, -76.260694}	3	5,9	{36.917132, -76.288053}	3	4,11,14	{36.784462, -76.182125}	3	2,10,15
4	{36.799725, -76.718446}	4	1,12,13,14	{36.771763, -76.582726}	2	11,14	{36.836193, -76.302793}	1	14	{36.763795, -76.293411}	2	14,15	{36.915004, -76.242664}	3	1,3,10	{36.771764, -76.179322}	2	12,13
5	{36.677697, -76.909886}	2	6,13	{36.776730, -76.530319}	3	12,13,15	{36.817628, -76.308194}	1	13	{36.763258, -76.340098}	4	2,11,12,13	{36.899016, -76.281221}	2	5,13	{36.813679, -76.111068}	1	3
6				{36.859765, -76.579596}	3	12,13,15	{36.806629, -76.370209}	2	11,15	{36.740007, -76.353297}	2	4,14	{36.924497, -76.314932}	3	3,9,15	{36.810047, -76.111050}	2	2
7										{36.785209, -76.367223}	2	10,14	{36.947484, -76.313393}	1	2	{36.813692, -76.098485}	1	1
8										{36.770121, -76.234409}	2	2,10	{36.912603, -76.251312}	1	15	{36.803265, -76.087427}	3	4,9,15
9										{36.692141, -76.208281}	3	11,14,15	{36.881062, -76.219937}	3	11,12,13	{36.799740, -76.091835}	2	13,14
10										{36.697077, -76.269005}	2	9,12	{36.833710, -76.249506}	3	11,13,14	{36.798535, -76.066924}	3	5,12
11										{36.785569, -76.244654}	2	4,13				{36.796532, -76.060478}	1	11
12										{36.827737, -76.414446}	3	10,11,15				{36.788357, -76.058526}	3	2,3,10
13										{36.776817, -76.371724}	3	1,5,15				{36.773764, -76.041493}	2	4,13
14										{36.794055, -76.230189}	3	1,3,14				{36.766369, -76.042851}	3	9,14,15
15																{36.755122, -76.009410}	3	1,5,11

Channel Assignment Solution The highlighted rows in Table III and columns in Table IV show the solutions to channel assignment for the PAL and GAA users, respectively. As shown in Table III, each PAL CBSD is assigned with channels belonging to $\mathcal{F}_P$, the lower 10 CBRS channels. Further, the number of channels allocated to a PAL CBSD in a county (the highlighted rows in Table III) meets its channel license requirement (the preceding rows). We also find that in the highlighted rows of Table III, all 10 channels in $\mathcal{F}_P$, except channel 6, have been assigned to different PAL CBSDs. In particular, channels 7 and 8, which are being used by incumbents, are also assigned to PAL holder 1's CBSD in county 7. This CBSD is physically located at {37.385214,-76.799968} (shown as a green square symbol in county 7 in Fig. 1) and is at a distance of 55 Km to its closed DPA point (DPA point 3), which is far away.

The highlighted columns in Table IV show our solutions to channel assignment for GAA users in the 13 counties. The optimal objective value for OPT-R (also OPT) is found to be 131.73. All 15 channels of the CBRS band $\mathcal{F}$ have

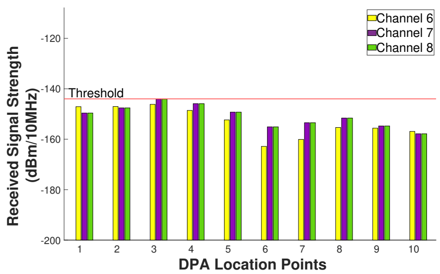

Fig. 3. Received aggregate PAL and GAA interference signal strengths at different DPA points.

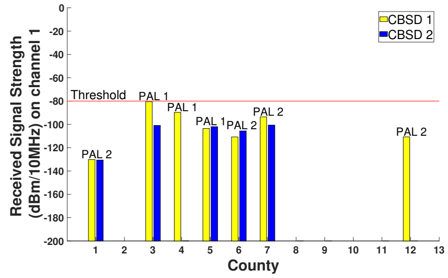

Fig. 4. Received highest aggregate interference signal strengths on channel 1 within the PPAs of PAL CBSDs.

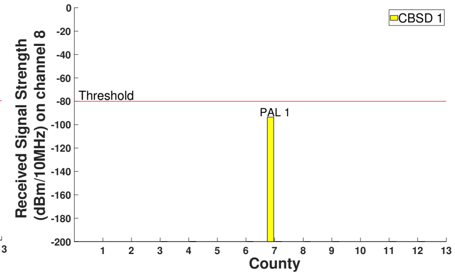

Fig. 5. Received highest aggregate interference signal strengths on channel 8 within the PPAs of PAL CBSDs.

been used in different counties, as shown in the highlighted columns of Table IV. Among the total 109 GAA users (each being independent from the others), 104 have been allocated with the number of channels to meet their full demand. By observing the channel allocation solution for the GAA users in the highlighted columns of Table IV, we can feel the force of our log-based objective function at work, which aims to addresses both fairness and efficiency simultaneously in channel allocation.

In Table IV, we find that a GAA user can operate on the same channel as a PAL holder in the same county, which maximizes spectrum efficiency. For instance, in county 3, channel 3 is assigned to PAL holder 2 (with only 1 CBSD) and GAA users 5, 8 and 12.

Further, GAA user 1 in county 7 is assigned with channels 7 and 8, both of which are used by PAL holder 1 and the incumbents. This GAA user 1 in county 7 is physically located at $\{37.425609,-76.812708\}$, which is represented by a red diamond symbol in county 7 in Fig. 1. This GAA user is at a distance of 59 Km from its closest DPA point (DPA point 3), which is far away.

Channel 6 (also assigned to the incumbents) has been assigned only to GAA user 5 in county 8. This GAA user is physically located at $\{36.677697,-76.909886\}$, which is represented by a red diamond symbol in county 8 in Fig. 1. It is at a distance of 54 Km from its closest DPA point (DPA point 3).

Interference Results With the channel allocation solution for PAL and GAA users highlighted in Tables III and IV, we now examine the interference levels on the incumbents and PAL holders. Let's start with the incumbents. Fig. 3 shows the aggregate interference (from PAL and GAA users) received at 10 DPA points on channels 6, 7, and 8. We find that received interference strength at all DPA points is smaller than -144 dBm/10MHz. This validates our solution to OPT-R and confirms that the incumbents are protected as we intended.

Now we examine interference on the PAL holders. Fig. 4 and Fig. 5 show the aggregate interference signal strength at a point on PAL holders' PPA that suffers from the highest interference on channel 1 and channel 8, respectively. Recall that channel 1 is not used by any incumbent while channel 8 is used by the incumbents. We see that the received interference

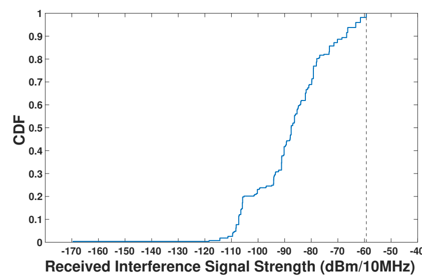

Fig. 6. CDF of aggregate interference signal on all channels among all GAA users.

strength at all the highest interference points within all the PPAs are below the -80 dBm/10MHz threshold. Note that this interference strength only considers co-channel interference from other PAL holders and all GAA users.

Note that in Fig. 5, there is only one PAL holder in county 7 that uses channel 8. This is due to the presence of incumbents on channel 8 and as a result, channel assignment to PAL holders on channel 8 is very limited. We further note that county 7 is the one that is farthest away from the 10 DPA points. In this county, the highest interference received within PAL holder 1 CBSD's PPA is from GAA user 1, which is also in county 7. The highest received interference signal strengths within the PPAs on other channels are similar to those shown in Fig. 4 and Fig. 5 and thus are not included here to conserve space.

Now we examine interference on the GAA users. Recall that unlike incumbents and PAL holders, there is no specific interference threshold requirement on the GAA users. To see what interference level each GAA user experiences on each channel, in Fig. 6, we plot the cumulative distribution function (CDF) of aggregate received interference strengths (from both PAL and other GAA users) for all GAA users over all allocated channels. We see the maximum interference strength is -59.31 dBm/10MHz and the median is -87.55 dBm/10MHz. In this figure, we find that, for this case study, 90% of channels used by GAA users experience an interference strength lower than -67 dBm/10MHz; 68.9% of channels used by GAA users experience an interference strength lower than -80 dBm/10MHz. Our results show that by including constraints

(17) and (18), the interference strengths are kept under control and the GAA service can offer a reasonable QoS. Note that if we further increase the distance separation of channel reuse in constraint (18) (e.g., by adding a tuning factor before $(R_n + R_g)$), we can make a controlled trade-off between the interference signal level experienced by the GAA users and the objective value (GAA's channel utilization).

C. Computation Time

The computational time to obtain the results in the last section is 22.44 seconds, which meets the timing requirement on a SAS [1]. When we further increase the number of GAA users to over 180 (while keeping the same number of PAL CBSDs), the computation time is getting close to 300 seconds—the allowed time limit for a SAS to obtain a channel allocation solution. For a larger scale of PAL and GAA users, we need to cut down computation time for the MILP. One promising approach is to employ GPU platform [16], whose potential and capability in solving large scale complex MILP problems in real time has been demonstrated in [17]–[19]. Exploration of real time computing is beyond the scope of this paper and we leave it for our future research.

V. Conclusions

In this paper, we developed a mathematical framework for channel allocation to PAL and GAA users in the presence of Navy shipborne radar incumbents. Our framework consists of rigorous model to characterize FCC's channel allocation rules and regulations for CBRS, with purposeful relaxation of channel and spatial contiguity requirement so as to maximize optimization space. By leveraging some novel reformulation techniques, we converted the original MINLP to a MILP and showed that there is no approximation error in the reformulation process. By applying real-world geographic data for a SAS along Virginia's east coast, we found that the solution to our MILP formulation meets all interference requirements of shipborne radar incumbents and PAL holders while achieving optimal channel utilization for GAA users. Further, the computation time met the allowed time limit for a SAS when performing channel allocation. This research lays the ground work for optimal channel allocation to PAL and GAA users in CBRS.

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